Math 2568, Quiz #11, Spring 2015 Name KEY

Instructions: Show all work. Some problems will instruct you to complete operations by hand, some can be done in the calculator. To show work on calculator problems, show the commands you used, and the resulting matrices. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. Find the similarity transformation, if it exists, of the matrix $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 5 & -2 \\ 1 & 3 & 1 \end{bmatrix}$ that is capable of diagonalizing the matrix.

$\lambda = 0$

The two remaining eigenvalues are not real. A can only be diagonalized w/ complex entries for both P and D.

$$(1-\lambda)\begin{vmatrix} 5-\lambda & -2 \\ 3 & 1-\lambda \end{vmatrix} - 2\begin{vmatrix} 2 & -2 \\ 1 & 1-\lambda \end{vmatrix} - 3\begin{vmatrix} 2 & 5-\lambda \\ 1 & 3 \end{vmatrix}$$

$$(1-\lambda)[(5-\lambda)(1-\lambda)+6] - 2[2-2\lambda+2] - 3[6-5+\lambda] = 0$$

$$(1-\lambda)[\lambda^2-6\lambda+11] - 2[4-2\lambda] - 3[1+\lambda] = 0$$

$$\lambda^2-6\lambda+11-\lambda^3+6\lambda^2-11\lambda-8+4\lambda-3-3\lambda = 0$$

$$-\lambda^3+7\lambda^2-16\lambda = 0$$

$$-\lambda(\lambda^2-7\lambda+16) = 0 \qquad \lambda = 0$$

$$\lambda = \frac{7 \pm \sqrt{49-64}}{2}$$ complex

2. Find the similarity transformation for the matrix $B = \begin{bmatrix} 0 & 5 \\ -2 & 2 \end{bmatrix}$ that converts this matrix into a similar rotation matrix. Then use that matrix to find the angle of rotation.

$-\lambda(2-\lambda)+10=0$

$\lambda^2-2\lambda+10=0$

$\lambda = \frac{2 \pm \sqrt{4-40}}{2} = \frac{2 \pm \sqrt{-36}}{2} = \frac{2 \pm 6i}{2} = 1 \pm 3i$

$C = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ for $\lambda = a - bi$ $\quad \lambda = 1 - 3i$ ($a = 1$, $b = 3$)

$C = \begin{bmatrix} 1 & -3 \\ 3 & 1 \end{bmatrix}$ $\quad r^2 = 1^2 + (-3)^2 = 10$

$C = \sqrt{10} \begin{bmatrix} 1/\sqrt{10} & -3/\sqrt{10} \\ 3/\sqrt{10} & 1/\sqrt{10} \end{bmatrix}$ $\quad \theta = \cos^{-1}(1/\sqrt{10}) = 1.2490$ in radians

or $71.6°$

$r \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$